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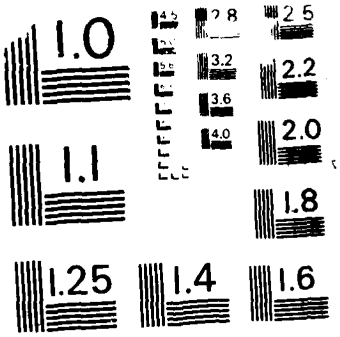

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APPLICATIONS OF SIGNAL PROCESSING

IN DIGITAL COMMUNICATIONS

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Our research activity during the period covered by this report was focused on the design of trellis codes. The idea is to match the code to the specific channel on which the digital transmission takes place. The development described here is based on the discovery, made by Calderbank and Mazo and recently published, that trellis codes can be described in a form which is basically the input-output relationship of a nonlinear system with memory.

Based on our previous research activity on modeling of nonlinear systems using Volterra series, we approached the code design problem as follows. We look at the encoder as at a nonlinear system whose aim is to <u>compensate</u> for the unwanted nonlinearities of the channel. As a result, the effect of coding is to remove, at least partially, certain undesired features of the transmission channel.

The paper herewith enclosed includes some preliminary results obtained along these lines. The paper was submitted for publication in the IEEE Journal on Selected Areas in Communications (special issue on Computer-Aided Modeling and Design of Communication Systems.)

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We consider the design of multidimensional signal sets and their combination with block or trellis codes. The goal is to achieve a high efficiency in the use of frequency spectrum for digital communications.

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ANALYSIS AND COMPENSATION OF NONLINEARITIES

IN DIGITAL TRANSMISSION SYSTEMS

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ABSTRACT

We consider the compensation of channel nonlinearities in digital radio communication systems. A discrete system with memory, inserted between the source and the modulator, is designed with the aim of providing an equivalent channel with a distortionless linear part and no nonlinearities up to a given order. This design is based on a Volterra series model of the channel, and on the theory of p-th order inverse systems.

Since the compensator design is based on a mathematical model of the channel, the problem of model identification is considered. A modeling technique is described, based on computer simulation and application of orthogonal Volterra series. Several examples show the performance of this class of compensators.

1. INTRODUCTION AND MOTIVATION OF THE WORK

As the demand for RF spectrum increases, high-speed data transmission over radio channels is likely to benefit from consideration of high-capacity modulation formats, such as multilevel quadrature amplitude modulation (QAM). Their application has been slowed down by the presence of amplitude (AM/AM) and phase (AM/PM) nonlinearities present in radio-frequency (RF) power amplifiers driven at or near saturation for better efficiency. Actually, the nonlinear distortions introduced by these amplifiers make the standard channel model, i.e., the additive white Gaussian noise channel, far from realistic, and hence system designs based on it far from optimum.

On the additive white Gaussian noise channel, it is well-known that QAM signals with a rectangular configuration provide a better bit-error rate (BER) performance than phase-shift keying (PSK) with an equal number of points. However, this situation seems to be reversed when nonlinear distortions are present in the channel. To cope with this problem, the simplest approach is to back off from the saturation point of the amplifier characteristics, in order to have the signal amplitude fluctuations to involve a range in which AM/AM characteristics are close to linear, and AM/PM characteristics close to a constant. However, this procedure results into a loss of power efficiency that might be unacceptable. In fact, in several instances it was verified (see for example [20]) that, as the number of energy levels in the signal constellation increases, the TWT working point should be backed off more to compensate for the nonlinear behavior of the amplifier. In this situation, it may occur that the beneficial effect of the increase in linearity is offset by the corresponding decrease of amplifier's output power. As a result, PSK (which has only one energy level) may

perform better than QAM (which has more) [20]. This is why more sophisticated solutions are called for.

From the above discussion, it appears rather natural to investigate two-dimensional signal constellations that outperform PSK, and yet do not suffer excessive degradations due to channel nonlinearities. This paper is devoted to this problem, through an approach that combines the choice of the modulation format and the compensation of channel nonlinearities.

The channel model on which our analysis will be based is time-discrete. We assume for simplicity that the modulated signal is sent through a nonlinear system with memory before being affected by additive white Gaussian noise at its output. In other words, the discrete channel consists of two separate parts: a noiseless deterministic part, and a noise adder. Traditionally, there are two philosophies intended to cope with the problem of channel nonlinearities. One of them consists in accepting the channel as is, without trying to do anything to modify its behavior, and to design the receiver so as to minimize the joint effects of intersymbol interference, nonlinearities, and noise. The most effective nonlinear signal processing technique based on this approach is maximum likelihood sequence estimation (MLSE), to be performed by the Viterbi algorithm [12],[14],[17,Chapter 10]. Unfortunately, this technique requires a processing complexity which may make it unsuitable for implementation at very high data rates. For this reason, suboptimum receiver schemes are attractive: among them, we can recall nonlinear equalization schemes [4], nonlinear cancellers [1],[2], optimum linear equalizers [15], and optimum linear receiving filters [16]. It must be kept in mind, however, that a fundamental limit to the performance of any of these receivers (and, more generally, of any conceivable receiver, either linear or nonlinear)

depends on the minimum Euclidean distance between the signals observed at the output of the noiseless (i.e., deterministic) part of the channel [18]. Stated in words, this limitation is due to the fact that the signal to be processed by the receiver is affected by noise: hence, any attempt to compensate for the channel distortion by introducing a sort of "inverse distortion" will enhance the noise. For this reason, it appears logical to investigate solutions based on the compensation of the nonlinearity <u>before</u> noise addition. This procedure should make the channel to look as similar as possible to a Gaussian channel.

If this approach is chosen, there are several factors and constraints that should be kept in mind. One of them is of course the ultimate performance that the nonlinearity compensation system can achieve. The second one is the ease of design, the implementation complexity, and the cost. The third one is that the compensator itself may expand the signal bandwidth, in spite of the fact that out-of-band emission must be kept under control [7]. In fact, while a predistorter reduces out-of-band emission after the amplifier, it may increase it before the amplifier. This can be a problem, for example in a satellite system with the predistorter located in the ground station to compensate for the on-board nonlinearity. Finally, in certain cases provision must be made for <u>adaptive</u> compensation: in fact, when a constellation with a large number of points is used by the modulator, even variations in amplifiers characteristics caused by temperature changes, dc power variations, and component aging can degrade the system performance [9]. Both analog and digital predistorters can in principle be considered: however, besides being more complex and expensive, and less flexible, the analog predistorters seem to perform worse than their digital counterparts [5]-[6]. Hence, consistently with our assumption of a discrete channel model, we shall consider digital predistortion.

2. PREDISTORTING THE SIGNAL CONSTELLATION

2.1 Memoryless predistortion

In this section we shall consider the action of a predistorter, i.e., a device placed in front of the channel and whose aim is to compensate for the unwanted effects of channel nonlinearities. Assume first that the channel has no memory (i.e., no bandwidth-limiting components exist) and consider the effect of a predistorter placed just before the nonlinear channel. In this situation, which we shall refer to as memoryless predistortion, the compensator acts by skewing the signal constellation in such a way that, when passed through the nonlinear device, it will resume the original shape (e.g., a rectangular 16-QAM structure). In other words, the compensator task is to invert the discrete transmission channel. This operation does not modify the spectrum, and hence the bandwidth occupancy, of the transmitted signal, but of course its effectiveness is critically dependent on the assumption that the channel has no memory.

2.2 Predistortion with memory. The p-th order inverse

Consider instead the more realistic assumption of a channel with memory. In this situation, the compensator is faced with a far harder task, the inversion of a nonlinear system with memory. Now, not all nonlinear systems possess an inverse. Also, many systems can be inverted only for a restricted range of input amplitudes [23,p.123 ff.]. However, it is always possible to define a p-th order inverse, for which the input amplitude range is not restricted.

The definition of p-th order inverse is based on a Volterra-series model of the discrete nonlinear channel (see [17] and the

references therein). This model provides an exceedingly general characterization of nonlinear systems with memory based on the so-called Volterra kernels, a set of parameters which can be thought of as the extension to the nonlinear case of the concept of impulse response of a linear channel. Given a nonlinear system H, its p-th order inverse is one that, when cascaded to H, results into a system in which the first-order Volterra kernel is a unit impulse, and the second through the p-th order Volterra kernels are zero [23]. In other words, if the p-th order nonlinear inverse channel is synthesized at the transmitter's front end, the compensated transmission channel will exhibit no linear distortion, and no nonlinear distortion up to order p. Obviously, the performance of the p-th order-compensated channel will depend on the effect of the residual distortions.

2.3 Memoryless predistortion vs. predistortion with memory

Before proceeding further with an analytical description of the compensation based on p-th order channel inversion, it is convenient to stop the discussion for a while, and provide an interpretation of the two types of predistorters described in the previous subsection. Memoryless predistortion is the operation of changing the (source) symbols a_n into the (channel) symbols $b_n=g(a_n)$, where $g(.)$ is a suitable complex function. If these modified symbols are viewed as a new signal set entering the channel (and matched to it), we may think of the compensator as being incorporated in the modulator. In conclusion, the design of a predistorter for a channel without memory is equivalent to the design of a new modulation scheme.

Consider then a predistorter with memory. Its operation consists of transforming the source symbols a_n into channel symbols b_n whose values depend not only on a_n but also on L previous symbols. Thus,

$$b_n = \Gamma(a_n, a_{n-1}, \quad \ldots \quad , \quad a_{n-L}). \tag{2.1}$$

If we define the <u>state of the compensator</u> at time n, and we denote it by σ_n:

$$\sigma_n = (a_{n-1}, \ \ldots \ , a_{n-L}), \tag{2.2}$$

we can also write

$$b_n = \Gamma(a_n, \sigma_n), \tag{2.3}$$

which shows explicitly the dependence of the channel symbols b_n on the state of the compensator. This "sliding block" representation of the compensator operation shows that the compensator itself is equivalent to a trellis encoder. (This equivalence was first proved by Calderbank and Mazo [24]). In conclusion, we can think of the design of a predistorter with memory as of the choice of a trellis code, made in order to compensate for the channel nonlinearity.

3. COMPENSATION BASED ON p-TH ORDER CHANNEL INVERSION

We start our discussion by considering the cascade of two nonlinear systems (for motivation's sake, the reader can view one of the two systems as the compensator, and the other one as the channel to be compensated.) We shall base our treatment of the subject on Volterra series representations of bandpass systems [17, Chapter 10], and we shall use, for notational simplicity, tensor notations, as suggested in [22]. These notations imply that any subscript occurring twice in the same term is to be summed over the appropriate range of discrete time. Thus, for example, we write $x_i y_i$ instead of $x_1 y_1 + x_2 y_2 + \ldots$.

3.1 Cascading bandpass nonlinear systems

Subject to certain regularity conditions, a bandpass *nonlinear system can be described by the input-output relationship*

$$y_n = \sum_{a=-\infty}^{\infty} h^{(1)}_{n-a}\, x_a + \sum_{a=-\infty}^{\infty}\sum_{b=-\infty}^{\infty}\sum_{c=-\infty}^{\infty} h^{(3)}_{n-a,n-b,n-c}\, x_a x_b x_c^*$$

$$+ \sum_{a=-\infty}^{\infty}\sum_{b=-\infty}^{\infty}\sum_{c=-\infty}^{\infty}\sum_{d=-\infty}^{\infty}\sum_{e=-\infty}^{\infty} h^{(5)}_{n-a,n-b,n-c,n-d,n-e}\, x_a x_b x_c x_d^* x_e^* + \cdots$$

$$= h^{(1)}_{n;a}\, x_a + h^{(3)}_{n;a,b,c}\, x_a\, x_b\, x_c^* + h^{(5)}_{n;a,b,c,d,e}\, x_a\, x_b\, x_c\, x_d^*\, x_e^* + \cdots . \tag{3.4}$$

From (3.4), it is seen that the system is characterized by the Volterra kernels $h^{(1)}_{n;a}$, $h^{(3)}_{n;a,b,c}$, ...

Consider now two bandpass nonlinear systems. Let the first (the compensator) be characterized by Volterra kernels f, the second (the channel) by Volterra kernels g, and denote by h the kernels of the system resulting from the cascade of the two. The first-, third-, and fifth-order h-kernels are explicitly given by

$$h^{(1)}_{n;a} = g^{(1)}_{n;v}\, f^{(1)}_{v;a} \tag{3.5}$$

$$\begin{aligned} h^{(3)}_{n;a,b,c} &= g^{(1)}_{n;v}\, f^{(3)}_{v;a,b,c} \\ &\quad + g^{(3)}_{n;v,w,z}\, f^{(1)}_{v;a}\, f^{(1)}_{w;b}\, f^{(1)*}_{z;c} \end{aligned} \tag{3.6}$$

and

$$\begin{aligned} h^{(5)}_{n;a,b,c,d,e} &= g^{(1)}_{n;v}\, f^{(5)}_{v;a,b,c,d,e} \\ &\quad + g^{(3)}_{n;v,w,z}\, f^{(1)}_{v;a}\, f^{(1)}_{w;b,c,d}\, f^{(3)*}_{z;e} \\ &\quad + g^{(3)}_{n;v,w,z}\, f^{(1)}_{v;a}\, f^{(3)}_{w;b,c,d}\, f^{(1)*}_{z;e} \\ &\quad + g^{(3)}_{n;v,w,z}\, f^{(3)}_{v;a,b,d}\, f^{(1)}_{w;c}\, f^{(3)*}_{z;e} \\ &\quad + g^{(5)}_{n;v,w,z,y,u}\, f^{(1)}_{v;a}\, f^{(1)}_{w;b}\, f^{(1)}_{z;c}\, f^{(1)*}_{y;d}\, f^{(1)*}_{u;e} \end{aligned} \tag{3.7}$$

It can be observed that (3.~~4~~) expresses a relationship 5 between first-order kernels which is nothing else but the discrete convolution of impulse responses of linear systems.

3.2 Volterra coefficients of p-th order compensator

Consider now p-th order compensation. Under the assumption that the linear part of system f, i.e., the linear functional determined by the first-order kernel of f, is invertible, it is possible to find a system g such that its cascade with f gives a system with no linear distortion, i.e.,

$$g^{(1)}_{n;v}\, f^{(1)}_{v;a} = f^{(1)}_{n;v}\, g^{(1)}_{v;a} = \delta_{n;a} \begin{cases} = 1 & n=a \\ = 0 & \text{elsewhere.} \end{cases} \tag{3.8}$$

This choice provides the first-order compensator. Eq. (3.8) *expresses nothing else* but the Nyquist criterion for the absence of intersymbol interference in the overall channel. In appearence, this sounds like a rather pleasant result, as it shows that even when dealing with a nonlinear channel the linear part must be designed (at least, if the "p-th order criterion" is accepted) to be Nyquist's. In the following, we shall see how the concept of "linear part of a channel" must be correctly interpreted.

The 3rd-order compensator is obtained by choosing $g^{(3)}$ so as to have $h^{(3)}=0$; by taking the discrete convolution of both sides of (3.6) with $g^{(1)}g^{(1)}g^{(1)*}$, and recalling (3.8), we get

$$g^{(3)}_{n;a,b,c} = - g^{(1)}_{n;y}\, f^{(3)}_{y;u,v,z}\, g^{(1)}_{u;a}\, g^{(1)}_{v;b}\, g^{(1)*}_{z;c} \tag{3.9}$$

The 5th-order compensator is obtained by choosing $h^{(5)}$ so as to have $h^{(5)}=0$; by taking the discrete convolution of both sides of (3.7) with $g^{(1)}g^{(1)}g^{(1)}g^{(1)*}g^{(1)*}$, and recalling (3.8), we get the

required $g^{(5)}$.

Before going further, let us consider a special situation (which is admittedly rather simplistic, but gives rise to considerations that might be interesting.) Assume that the channel nonlinearity is the cascade of a linear system L and a memoryless device D. The p-th order compensator for this channel can be easily computed, providing a result which matches intuition. In fact, it is the cascade of a linear filter, the inverse of L (say, L^{-1}), preceeded by a nonlinear memoryless device, the p-th order inverse of D. Notice that the cascade of L^{-1} and L gives rise to a Nyquist filter. This result shows that one way to compensate for the channel nonlinearity in this case consists of removing the channel memory and compensating for the resulting memoryless nonlinearity by memoryless predistortion.

A more realistic model, suitable as an approximation to a number of single-channel digital satellite comunication systems, assumes that the linear part of the channel has already been compensated by a suitable combination of channel filtering and linear equalization at the receiver's front-end. In this situation some simplifications arise. In particular we get, for the first- and third-order compensators:

$$g^{(1)}_{n;a} = \delta_{n;a}$$
$$g^{(3)}_{n;a,b,c} = - f^{(3)}_{n;a,b,c} \tag{3.10}$$

3.3 <u>The effect of compensation on power spectrum</u>

We consider now the effect of a p-th order compensator on the signal power density spectrum. The continuous-time signal at the

modulator output can be given the form

$$x(t) = \sum_{n=-\infty}^{\infty} b_n\, s(t-nT) \tag{3.11}$$

where (b_n) is the sequence of channel symbols, T is the symbol period (equivalently, T^{-1} is the baud rate), and s(t) is the basic waveform used by the modulator. The power density spectrum of signal (3.11) is given by (see [17, p.33])

$$G(f) = \frac{\beta_o}{T} |S(f)|^2 \left\{ \sum_{n=-\infty}^{\infty} \beta_n \exp(-j2\pi fnT) \right\} \tag{3.12}$$

where β_n is the autocorrelation of the symbol sequence at the compensator output and S(f) is the Fourier transform of s(t). It is easily recognized that the brackets in the RHS of (3.12) contain the discrete Fourier transform of the sequence (β_n), i.e., the power density spectrum of the sequence at the compensator output. This is a periodic function of f with period 1/T.

From (3.12), we see that the spectrum shaping effect of the compensator can be analyzed by evaluating the autocorrelation sequence (β_n). For example, a linear compensator responding to the source symbol sequence (a_n), $E|a_n|^2=1$, with the sequence (a_n+Aa_{n-1}), A a constant, will cause a spectral shaping $(1+A^2)+2A\cos(2\pi fT)$. A fact that might be unexpected a priori is that the <u>nonlinear</u> terms of the compensator can be irrelevant in shaping the spectrum. Consider, as an example, the compensator output $a_n+Aa_{n-1}+Ba_na_{n-1}a^*_{n-2}$. By direct calculation, it can be seen that $\beta_0=(1+A^2+B^2)$ and $\beta_{-1}=\beta_1=A$, while $\beta_i=0$ for $|i|\geq 2$. Hence, the third-order nonlinearity has, for $A^2 \gg B^2$(as is usually

the case when relatively mild nonlinearities must be compensated), very little effect.

3.4 Computing the linear part of the compensator

The computation of the linear part of the compensator, i.e., of the kernels $g^{(1)}$ that solve (3.8), deserves some further attention. By rewriting explicitly (3.8), we have

$$\sum_{n} f^{(1)}_{k-n}\, g^{(1)}_{n} = \delta_{k;0} \tag{3.13}$$

where δ denotes the Kronecker symbol. Since we are interested in a finite-complexity compensator, we consider a (perhaps approximate) solution of (3.13) which includes just a finite number of terms in the summation. Thus, our problem is equivalent to the design of a zero-forcing equalizer of finite length. Two technical assumptions are necessary here, namely, that there exists only a finite number of nonzero $f^{(1)}$-kernels, and that the polynomial whose coefficients are these kernels has no root with unit magnitude. Under these conditions, a solution exists for the kernels $g^{(1)}$ with values that decrease in magnitude away from a "center kernel". The procedure for computing these kernels, which requires finding the roots of a polynomial and the solution of a set of linear equations, can be found in [27].

4. THE ROLE OF CHANNEL MODELING - ORTHOGONAL VOLTERRA SERIES

From our preceding discussion it is seen that the compensator design is based on a Volterra-series model for the nonlinear transmission channel. Thus, the availability of such a model is crucial. As, apart from some very simple cases, analytical evaluation of Volterra coefficients is not feasible, computational techniques should be used. Basically, two approaches are available, which we shall refer to as "block modeling" and "global identification".

Consider first block modeling. It is based on a model of the channel as a cascade of linear, time-invariant filters and bandpass nonlinear devices whose input-output relationships are given in the form of Taylor series. In this case, the Volterra kernels are evaluated by combining the input-output relationships of the building blocks that form the channel (see, for example, [19]). Although this approach is apparently simple and straightforward, particularly when the channel itself is composed by a reduced number of blocks, in its application some care must be exercised by taking two important points into consideration. First of all, in many cases the number of nonzero Volterra coefficients is so large that the number of computations involved in evaluating higher-order coefficients may be impractical. The second one is more subtle. Perhaps the most important fact to be kept in mind when considering the identification of the channel is that nonlinear systems behave differently for different input signals. To understand the consequences of this statement, consider a simple example. Assume we are dealing with a channel responding to the input sequence x_n with the sequence $y_n=\alpha x_n+\beta x_n^3$, and assume that x_n can take only the values +1 or -1. Under these conditions, as $x_n=x_n^3$, the system behaves as <u>linear</u>, with input-output relationship $y_n=(\alpha+\beta)x_n$. On the contrary, if the input sequence can take values -3, -1, 1, and 3, the system really behaves as nonlinear. Hence, we realize that in a

Volterra series model each one of the nonlinear terms affect the transmitted sequence differently if different modulation formats are used. As an example, the third-order Volterra kernel $h^{(3)}_{011}$ has a different behavior on PSK and QAM signals. In fact, this kernel multiplies a term

$$b_n b_{n-1} b^*_{n-1}.$$

For PSK $b_{n-1}b^*_{n-1} = |b_{n-1}|^2$ = constant, and hence the kernel contributes to linear distortion only. As a conclusion, the Volterra kernels should be rearranged, after computation, to account for effects like this. Besides operating by inspection, a general way to reduce the Volterra coefficients in order to account for the modulation format at hand is based on Orthogonal Volterra Series. We shall consider this point further on.

Consider then global identification. This is entirely based on computer simulation, and consists of identifying the Volterra kernels of the transmission system (already in their reduced version) through a gradient algorithm (see [17, Chapter 10] for further details about the identification algorithm). Using global identification, the reduction problem mentioned above can be solved at once by using what we call Orthogonal Volterra Series, a type of expansion that depends on the channel input characteristics and does not need any further reduction.

Underlying theory

The Volterra expansion (3.4) has the structure of a Taylor series, and as such shares with Taylor series some negative features. For example, it might be inadequate to represent highly nonlinear systems, or, equivalently, nonlinear systems with large outputs. Moreo-

ver, the Volterra model of a given channel may not be improved by adding more terms to the series. Finally, even when the channel input sequence are independent random variables, the terms in (3.4) are not even uncorrelated. Now, many of the drawbacks of Volterra series can be fixed up by using orthogonal polynomial expansions. These consist of using an input-output relationship of the type

$$y_n = h^{(1)}_{n;a}\, Q^{(1)}(x_a) + h^{(3)}_{n;a,b,c}\, Q^{(3)}(x_a, x_b, x_c) + \dots \tag{4.1}$$

where $Q^{(i)}$ denotes a polynomial of degree i that is orthogonal with respect to the sequence of random variables (x_n). More precisely, the expectation $E[Q^{(i)}Q^{*(j)}]$ is equal to zero whenever $i \neq j$, or $i=j$ but the arguments of $Q^{(i)}$ and $Q^{(j)}$ are not a permutation of each other. If it is assumed that the sequence (x_n) is a stationary sequence of independent, identically distributed random variables, the construction of these orthogonal polynomials is a relatively straightforward task. In fact, the resulting polynomials turn out to be a generalization of multidimensional Hermite polynomials, as defined by Grad [25]. They can be constructed, by using an observation of Zadeh [26], according to the following rule:

$$Q^{(i)}(x_{k1}, x_{k2}, \dots, x_{ki}) = P_{n1}(x_{k1}) \cdots P_{ni}(x_{ki}) \tag{4.2}$$

where $n_1, \dots, n_i$ denote the number of indexes of the arguments of $Q^{(i)}$ equal to $k_1, \dots, k_i$, respectively, and $P_j(\cdot)$ are polynomials in a single indeterminate orthogonal with respect to the random variable x_n, i.e.,

$$E[P_i(x_n)P_j(x_n)] = 0 \quad \text{for } i \neq j,$$

where $E[\cdot]$ denotes expectation with respect to x_n. For example,

$$Q^{(3)}(x_1, x_1, x_3) = P_2(x_1)\, P_1(x_3).$$

Consider then the problem of constructing the polynomials $P(\cdot)$. They can be found using a procedure based on the selection of a sequence of linearly independent monomials in the variable x_n, say f_0, $f_1, \ldots$. Explicit formulas are

$$P_k(x_n) = \det \begin{bmatrix} f_k & f_{k-1} & \cdots & f_0 \\ E[f_{k-1}f_k^*] & E[|f_{k-1}|^2] & \cdots & E[f_{k-1}f_0^*] \\ & & \cdots & \\ & & & \\ E[f_0 f_k^*] & E[f_0 f_{k-1}^*] & \cdots & E[|f_0|^2] \end{bmatrix}$$

Application to digital radio modulation systems

In our situation, we can start from the sequence of monomials

$$1 \; , \; x_n, \; x_n^* \; , \; |x_n|^2, \; x_n^2 \; , \; x_n^{*2} \; , \; x_n^3 \; , \; |x_n|^2 x_n \; , \; |x_n|^2 x_n^* \; , \; x_n^{3*} \; , \; \ldots$$

This sequence must be reduced by taking into account the particular type of modulation scheme involved, which may render some of the monomials linearly dependent. For example, with unit-energy PSK we have $|x_n|^2 = 1$, and consequently the fourth, the eighth, and the ninth monomials above must be deleted from the list. Furthermore, for 4-phase PSK we have $x^3{}_n = \pm x^*{}_n$, which causes the seventh and the tenth monomial to be deleted, too.

Finally, the polynomials $Q^{(i)}$ associated with the particular modulation scheme can be constructed as follows. Use first rule (4.2), then delete the polynomials which correspond to the components of the channel output falling outside of the bandwidth of interest (see [17,pp. 542 ff] for further details). In practice, this correspond to keeping only the terms of the type x, xxx^*, $xxxx^*x^*$, etc. For example, the Q-polynomials for unit-energy PSK are, up to order three:

$$x_i \; , \; x_i x_j x_k^* \; , \; x_i^2 x_j^* \; .$$

Similarly, for unit-energy 16-QAM we have

$$x_i \; , \; x_i x_j x_k^* \; , \; x_i^2 x_j^* \; , \; [|x_i|^2 - 1]x_j \; , \; |x_i|^2 x_i - 1.32\, x_i \; .$$

5. SOME EXAMPLES OF APPLICATION

We shall now consider some examples of applications of the concepts outlined in previous sections. Examination of a few simple situations will allow us to show the applicability of this theory, and will hopefully enhance its comprehension.

We deal with a nonlinear channel modeled using a bandpass orthogonal Volterra series whose coefficients for PSK signaling are given in [17, p.566]. This channel results from the cascade of a rectangular shaping filter, a fourth-order Butterworth filter with 3-dB bandwidth 1.7/T (T is the signaling period), a typical TWT amplifier exhibiting both AM/AM and AM/PM conversion, and a second-order Butterworth filter with 3-dB bandwidth 1.1/T. The amplifier is driven at saturation when the sequence at the input of the discrete channel has magnitude 1. (See [17, Chapter 10], for more details about this channel.) We proceed to compensate for this channel by inserting in

front of it a nonlinear device with memory obtained as an approximation of th channel inverse. In particular, we denote by $(r_1, r_3, \ldots, r_p)$ the compensator obtained by retaining in it only r_1 first-order Volterra coefficients, r_3 third-order coefficients, etc. Thus, for example, (3,1) indicates a third-order compensator with three first-order and one third-order coefficient. The coefficients are chosen whose indexes are the same as the Volterra coefficients of the channel having the largest magnitudes. Our computational experience has shown this choice to be the most effective, although no formal proof of its optimality has been obtained yet.

Consider first transmitting an 8-PSK symbol sequence driving the amplifier at saturation. The reduced Volterra kernels are listed in Fig.1. The symbols are $\exp(j0)$, $\exp(j\pi/4)$, ... , $\exp(j7\pi/4)$. Without any compensation, the samples of the received signal form the constellation shown in Fig. 2, where only the first quadrant is shown for sake of clarity. If a (1,1) compensator is used, the corresponding constellation looks like in Fig. 3. The reduction in the constellation spread is apparent. Notice also the phase rotation introduced, which compensate for the rotation caused by the amplifier's AM/PM. A (4,1) compensator gives the result shown in Fig. 4, while the effect of a (4,5) compensator is depicted in Fig. 5.

For 16-QAM signals with the highest energy level driving the amplifier at saturation, the channel quality without compensation is even less satisfactory. Fig. 6 shows the received constellation in the first quadrant: it is seen that the two clusters overlap. The effect of a (1,1) compensator is shown in Fig. 7, while Fig. 8 depicts the effect of a (4,1) compensator. Similar results have been obtained for 16-PSK: see Fig. 9 (uncompensated channel), Fig. 10 ((1,1) compensator), Fig. 11 ((4,1) compensator), Fig. 12 ((4,5) compensator).

For all these situations, the effect of the compensator on the power density spectrum has been evaluated, and found to be practically irrelevant.

Consider then the case of a channel whose linear part has been designed to satisfy the Nyquist criterion for no intersymbol interference. Specifically, assume the transmitter and receiver filters to have the common shape of a square-root raised cosine, with a rolloff factor 0.5. The channel between them is modeled through a nonlinear amplifier exhibiting AM/AM and AM/PM conversion effects, driven at saturation, and whose input-output characteristics are described using a model due to Saleh (see [28, Eqs. (1)-(5)]) with parameters

$$\alpha_a = 1.9638 \qquad \alpha_\varphi = 2.5293$$
$$\beta_a = 0.9945 \qquad \beta_\varphi = 2.8168.$$

Block identification of this channel turns out to provide rather disappointing results. For example, we get a center linear kernel whose value is $h_0^{(1)} = 1.97 + j\ 0.08$, which fails to account for the rotation (about 40^{o}) introduced by the amplifier at its saturation point. We need to reduce the Volterra expansion obtained by block identification, or, even better, to use global identification and orthogonal polynomials. This operation provides the coefficients for the orthogonal Volterra series. The largest among them are listed, up to order three, in Fig.13. It can be seen that the central linear coefficient reflects the phase rotation caused by the nonlinear amplifier. Figs. 14 and 15 provide a comparison among the scattering diagrams of 8-PSK and 16-PSK, respectively, at the output of a channel with (1,0) compensation (i.e., compensated only for the rotation and the amplitude scaling) and (3,6) compensation. Inspection of these scattering diagrams shows that the effect of the third-order compensator, although evident, is less dramatic than for the cases considered previously.

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FIGURE CAPTIONS

Fig.1 - A set of Volterra kernels for a PSK channel.

Fig.2 - Signal constellation at the output of the channel of Fig.1 when 8-PSK is used.

Fig.3 - Same as in Fig. 2, with a (1,1) compensator.

Fig.4 - Same as in Fig. 2, with a (4,1) compensator.

Fig.5 - Same as in Fig. 2, with a (4,5) compensator.

Fig.6 - Signal constellation at the output of the channel of Fig.1 when 16-QAM is used.

Fig.7 - Same as in Fig. 6, with a (1,1) compensator.

Fig.8 - Same as in Fig. 6, with a (4,1) compensator.

Fig.9 - Signal constellation at the output of the channel of Fig.1 when 16-PSK is used.

Fig.10- Same as in Fig. 9, with a (1,1) compensator.

Fig.11- Same as in Fig. 9, with a (4,1) compensator.

Fig.12- Same as in Fig. 9, with a (4,5) compensator.

Fig.13- A set of orthogonal Volterra series coefficients.

Fig.14- Signal constellations at the output of the channel modeled by the coefficients of Fig.13 for 8-PSK.

(a) With (1,0) compensation.

(b) With (3,6) compensation.

Fig.15- Signal constellations at the output of the channel modeled by the coefficients of Fig.13 for 16-PSK.

(a) With (1,0) compensation.

(b) With (3,6) compensation.

LINEAR PART
$f_0^{(1)} = 1.22 + J\ 0.646$
$f_1^{(1)} = 0.063 - j\ 0.001$
$f_2^{(1)} = -0.024 - j\ 0.014$
$f_3^{(1)} = 0.036 + j\ 0.031$
3RD ORDER NONLINEARITIES
$f_{002}^{(3)} = 0.039 - j\ 0.022$
$f_{330}^{(3)} = 0.018 - j\ 0.018$
$f_{001}^{(3)} = 0.035 - j\ 0.035$
$f_{003}^{(3)} = -0.04 - j\ 0.009$
$f_{110}^{(3)} = -0.01 - j\ 0.017$
5TH ORDER NONLINEARITIES
$f_{00011}^{(5)} = 0.039 - j\ 0.022$

Fig. 1

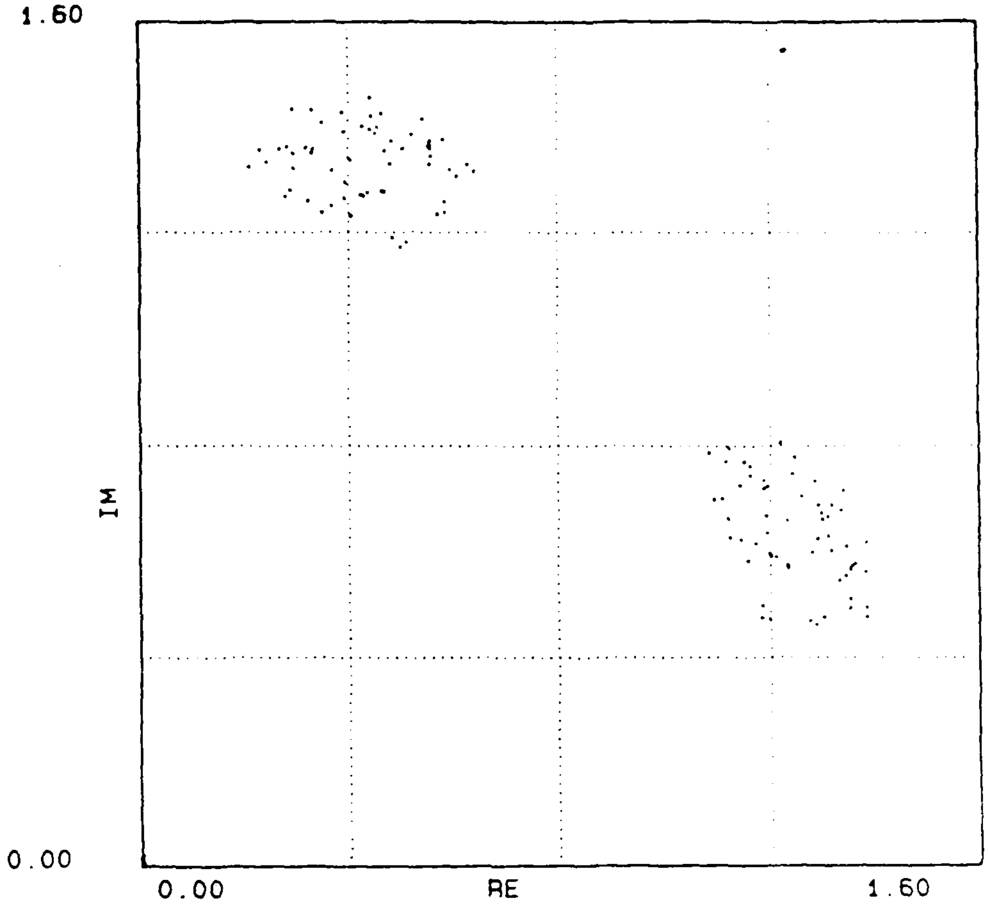
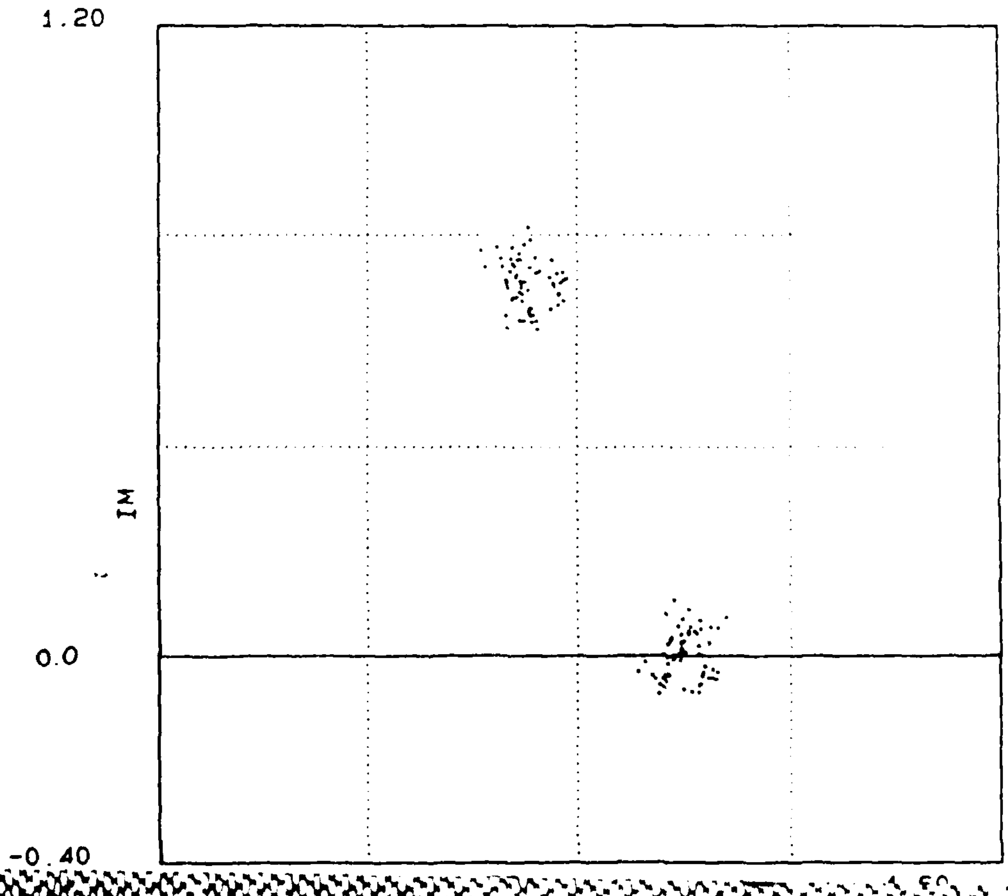


Fig 2



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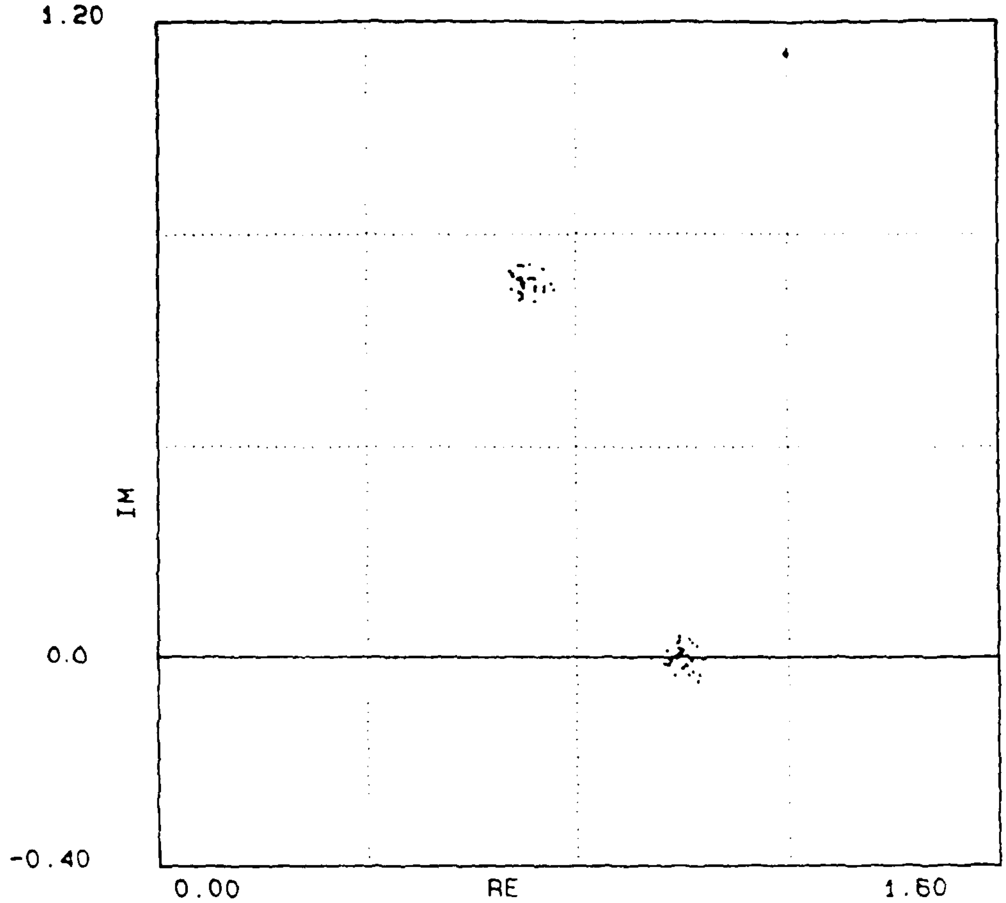

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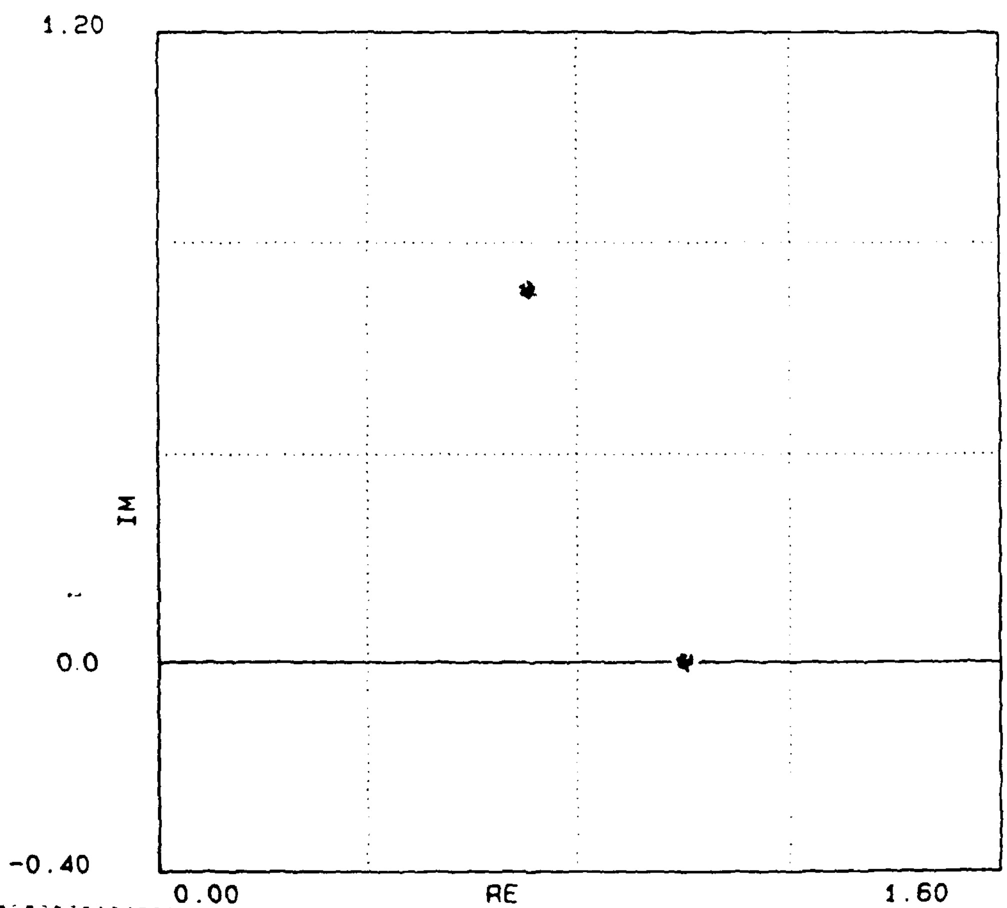

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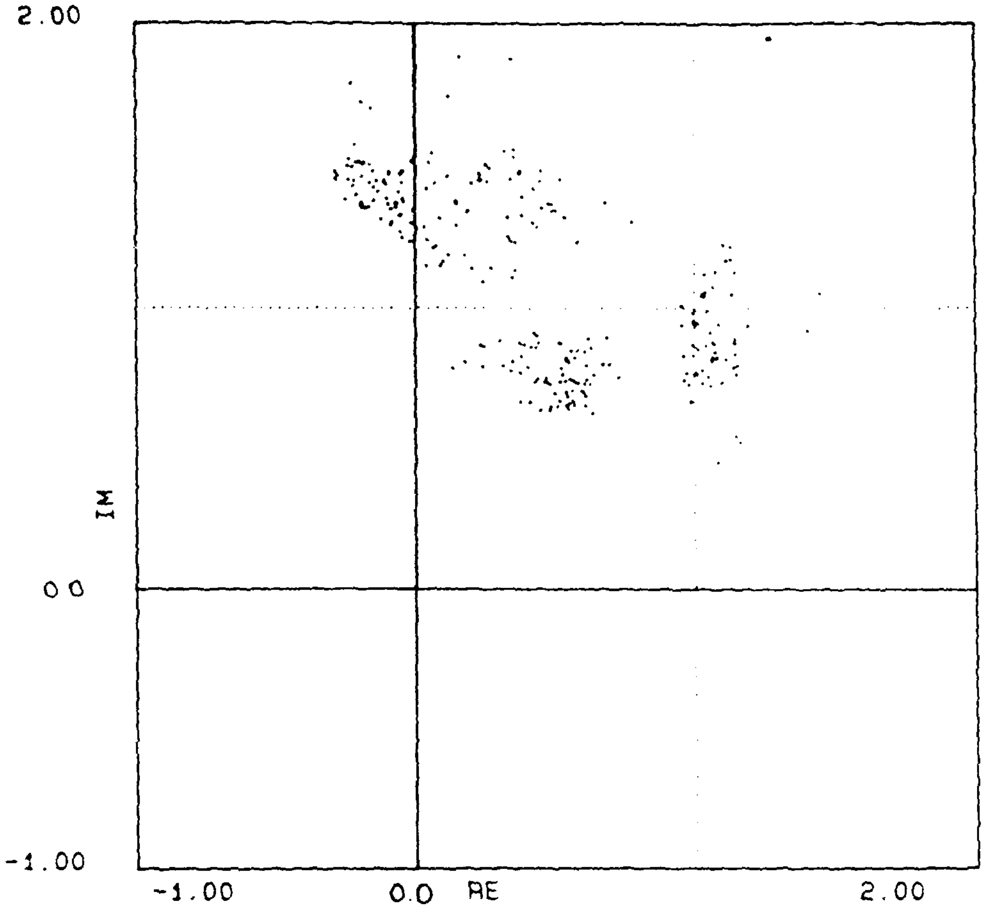


Fig 6

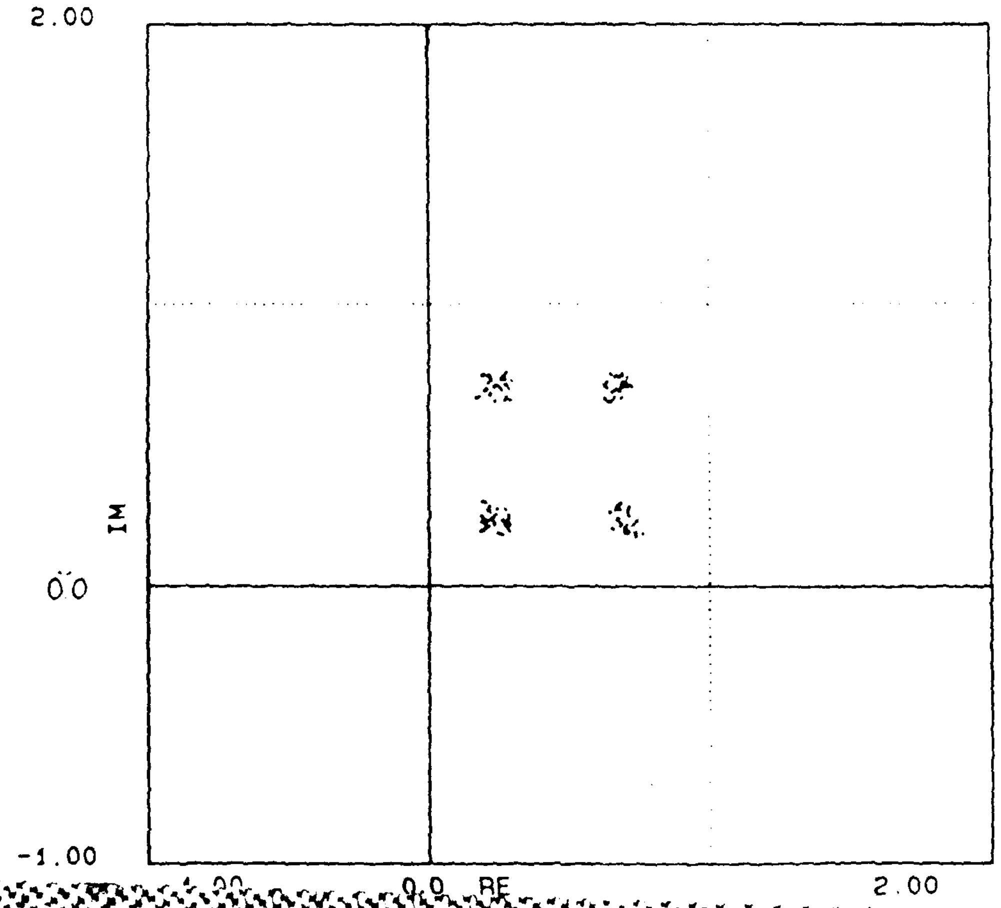


Fig 7

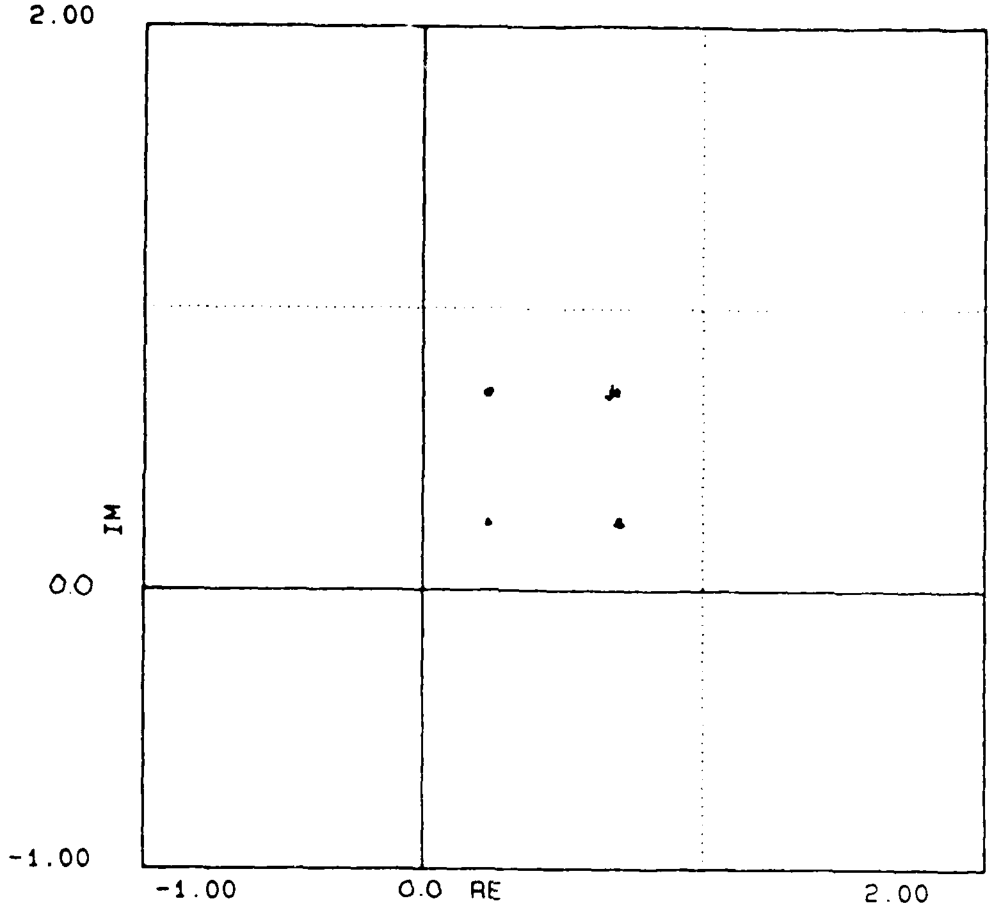


Fig. 8

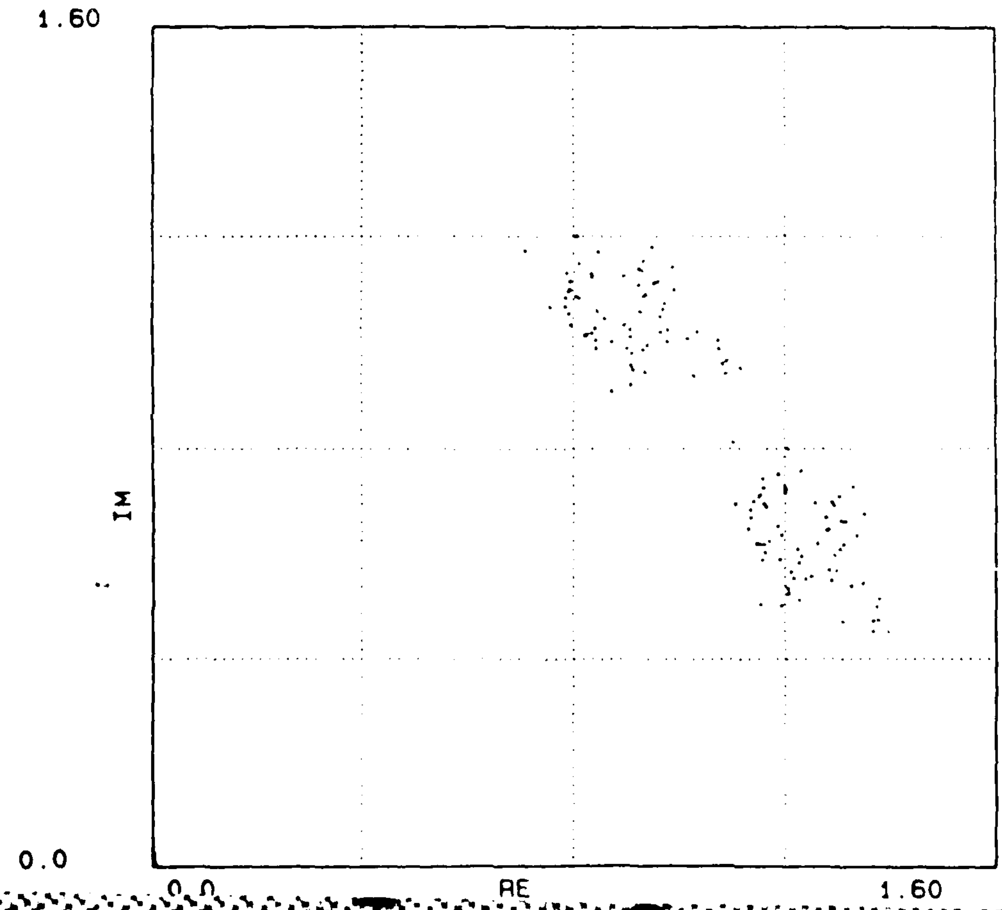


Fig 9

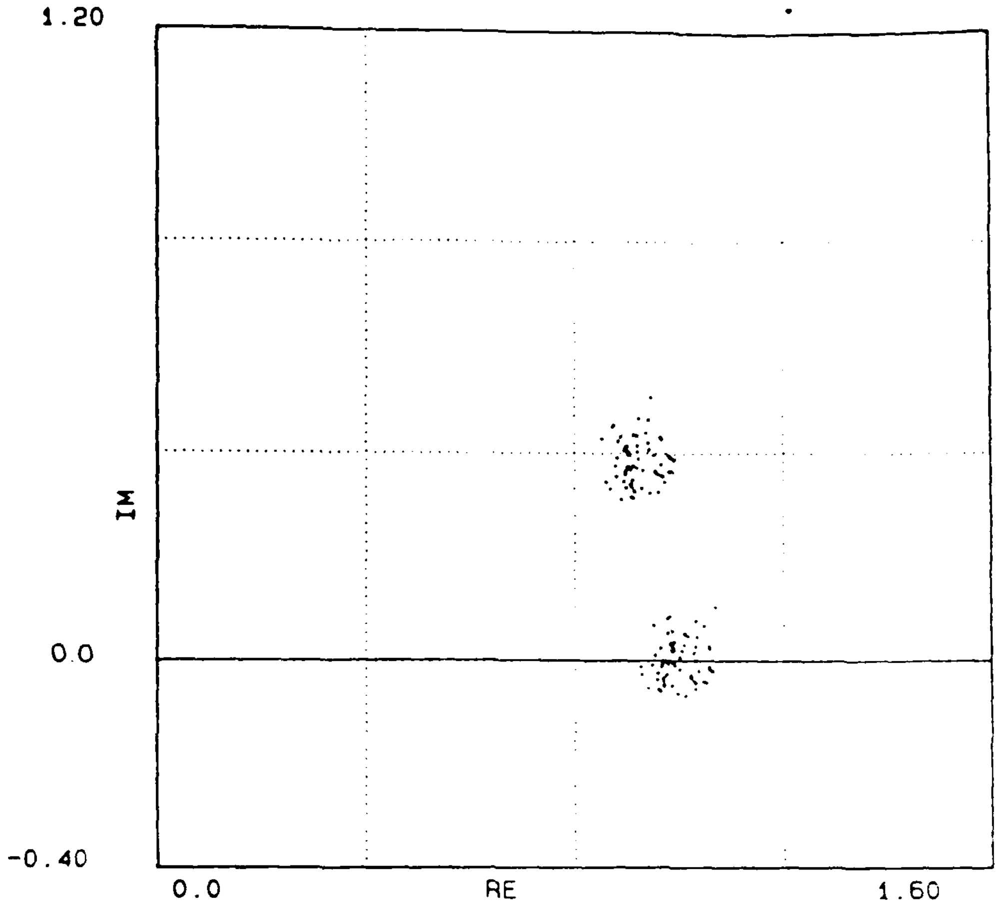


Fig. 10

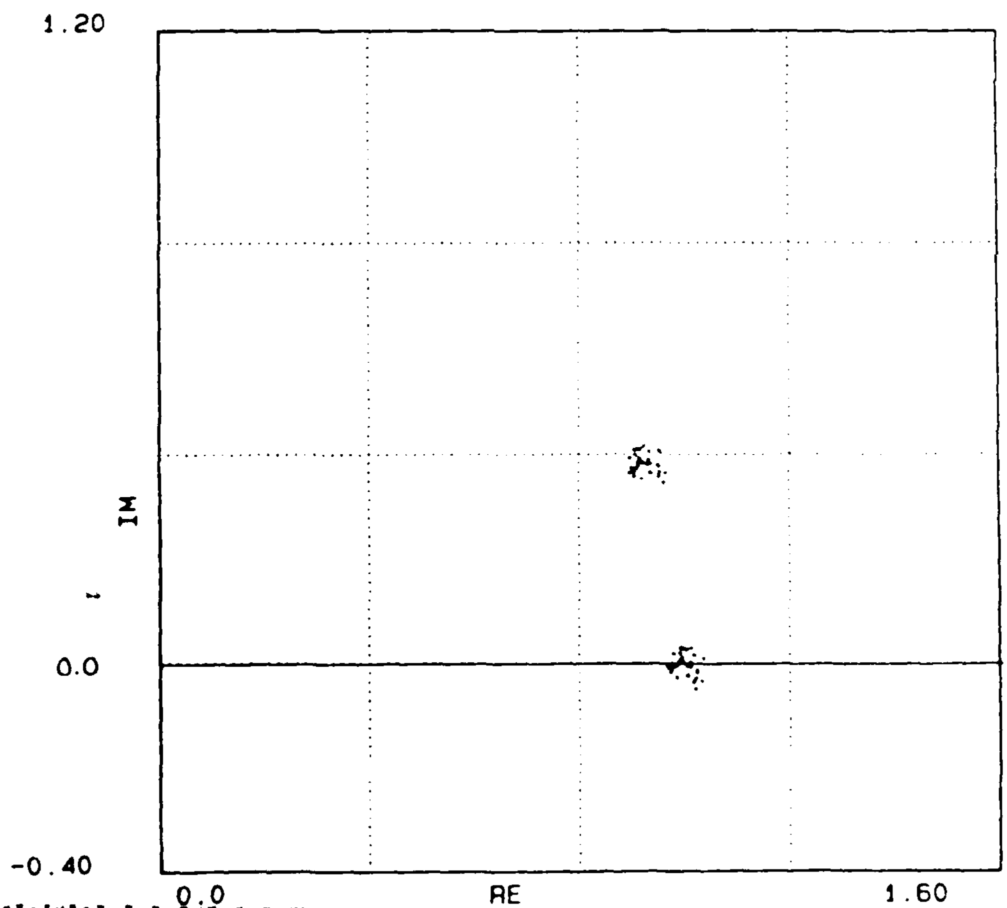


Fig. 11

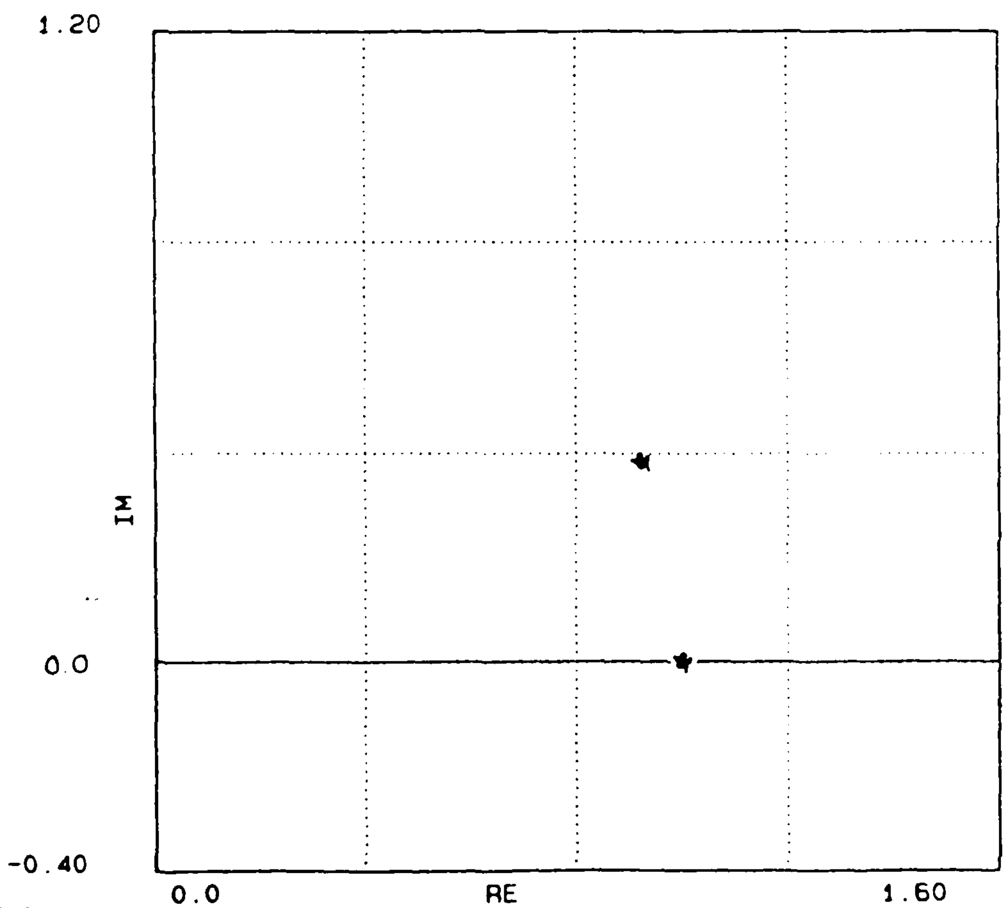
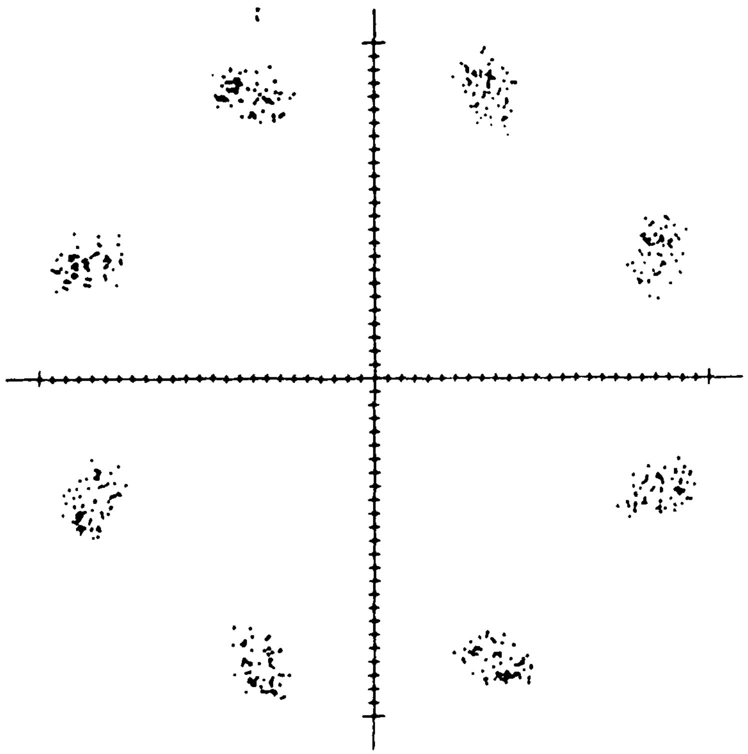


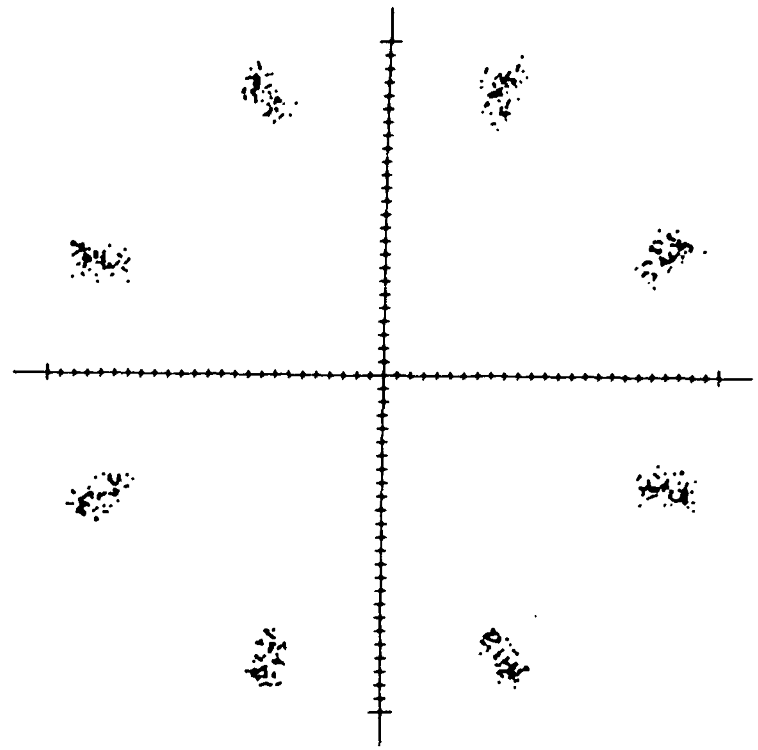
Fig 12

LINEAR PART	
$h_{-2}^{(1)}$	$= -0.01$
$h_{-1}^{(1)}$	$= 0.02 + j\,0.01$
$h_{0}^{(1)}$	$= 0.73 + j\,0.57$
$h_{1}^{(1)}$	$= 0.02 + j\,0.01$
$h_{2}^{(1)}$	$= -0.01$
3RD ORDER NONLINEARITIES	
$h_{-1-10}^{(3)}$	$=-0.02 - j\,0.01$
$h_{110}^{(3)}$	$=-0.02 - j\,0.01$

Fig. 13

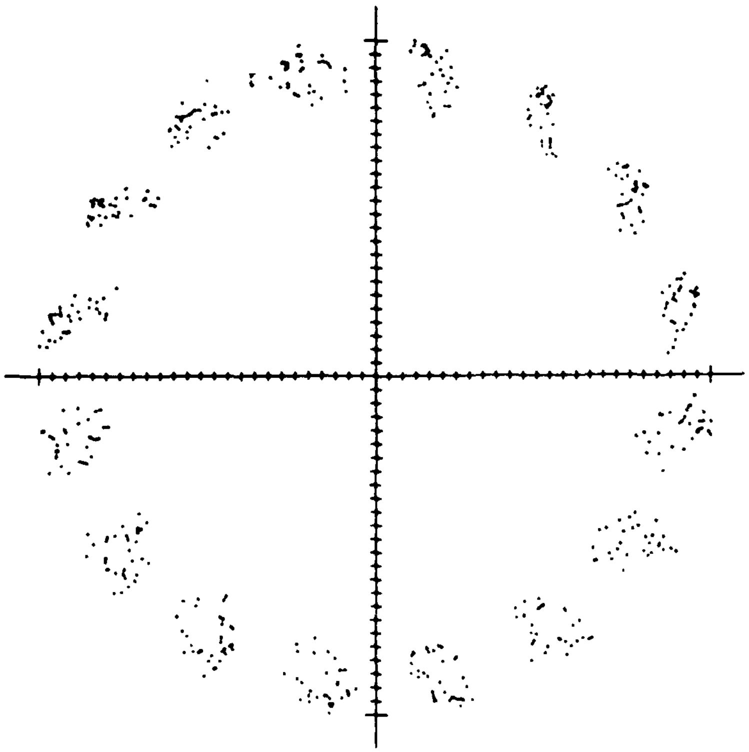


(a)

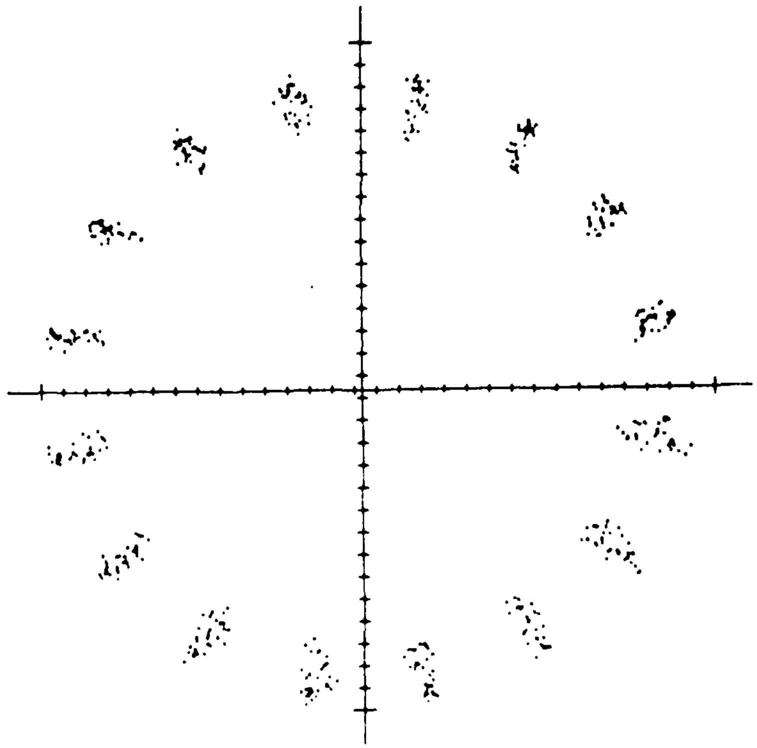


(b)

Fig. 14



(a)



(b)

Fig. 15

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